Resource Allocation in Cooperative Networks: the Rule of Games

Guoliang Xue
Arizona State University

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Outline

Basics
- Cooperative Communication
- Game Theory
- Auction Theory

HERA
- Motivation
- Optimal Assignment
- Payment of the Source Nodes
- Payment to the Relay Nodes

TASC
- Motivation
- Design
- Example
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Cooperative Communication

\[ C_R = f(s, r, d) \]

\[ (x_r, y_r) \text{ and } P_r \]

\[ s \leftrightarrow r \quad r \rightarrow d \quad s \leftrightarrow r \quad r \rightarrow d \quad \ldots \]

time

J.N. Laneman  D. Tse  G.W. Wornell
Multi-Source Cooperative Communication

![Diagram of a multi-source cooperative communication network]

\[ C(s_i, r_j) = \frac{C_R(s_i, r_j)}{n_j} \]

- \( r_j \) is the relay node assigned to source \( s_i \).
- \( n_j \) is the number of source nodes sharing relay node \( r_j \).
Game Theory

- **Player**: Entities in the game
- **Strategy**: Actions taken by players
- **Utility**: Valuation of players on the outcome of the game
Game Theory

**Best Response**
- The strategy which maximizes the player's utility, when other players' strategies are given

**Nash Equilibrium**
- Every player is playing its best response.

**Strongly Dominant Strategy**
- Every player is playing a strategy which produces a larger utility than any other strategy, regardless of other players' strategies
Auction Theory

- **Buyer**
  - Bidder buying service

- **Seller**
  - Bidder selling service

- **Bid**
  - Price provided by a buyer

- **Ask**
  - Price provided by a seller

- **Valuation**
  - True price that a bidder wants to bid or ask
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System Model

- $n$ source-destination pairs, denoted by $S$
- $m$ relay nodes, denote by $R$
- Each source-destination pair is assigned at most ONE relay node [Zhao et al. ISIT’06]
- Single radio
- Half duplex
- Interference: enough orthogonal channels
Challenges

**System Performance**
- Capacity depends on the relay assignment

**Selfishness**
- Wireless devices belong to independent entities
- Source nodes select relays solely to maximize their own utility

**Cheating**
- Relay assignment is based on the reported power from relay nodes
- Relay nodes can rig the assignment
HERA: An Integrated Optimal Relay Assignment Scheme

Selfish users converging to the optimal assignment

Selfish users being honest

Budget-balanced

Dejun Yang, Xi Fang, and Guoliang Xue; HERA: An Optimal Relay Assignment Scheme for Cooperative Networks; accepted for publication in IEEE Journal on Selected Areas in Communications, 2011

figure source: http://www.openclipart.org
Related Work

- Relay assignment to maximize the minimum capacity [Shi et al. Mobihoc’08]
  - A different objective
  - A constrained model

- Relay assignment to maximize the total capacity [Zhang et al. WCNC’09]
  - The same objective
  - A constrained model
Related Work

- Resource trade between source nodes and relay nodes [Wang et al. TMC’09, Huang et al. JSAC’08, and Zhang et al. ETRI’09]
  - No system performance guarantee
  - Only consider selfish behavior, not cheating behavior
### Motivating Example

<table>
<thead>
<tr>
<th></th>
<th>s₁</th>
<th>s₂</th>
<th>s₃</th>
<th>s₄</th>
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<tbody>
<tr>
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<td>4</td>
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**Total Capacity = 17**

<table>
<thead>
<tr>
<th></th>
<th>s₁</th>
<th>s₂</th>
<th>s₃</th>
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<td>3</td>
<td>1</td>
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<td>7</td>
<td>6</td>
<td>6</td>
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**Total Capacity = 20**

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<th>s₃</th>
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</table>

**Total Capacity = 24**

<table>
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<tr>
<th></th>
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<th>s₂</th>
<th>s₃</th>
<th>s₄</th>
<th>s₅</th>
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<tbody>
<tr>
<td>r₀</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>1</td>
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<tr>
<td>r₁</td>
<td>10</td>
<td>7</td>
<td>6</td>
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<td>r₂</td>
<td>4</td>
<td>8</td>
<td>4</td>
<td>10</td>
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</tr>
</tbody>
</table>

**Total Capacity = 22.5**
### Local Optimum vs. Global Optimum

**Local Optimum**

<table>
<thead>
<tr>
<th></th>
<th>s1</th>
<th>s2</th>
<th>s3</th>
<th>s4</th>
<th>s5</th>
</tr>
</thead>
<tbody>
<tr>
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<td>4</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>1</td>
</tr>
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<tr>
<td>r2</td>
<td>4</td>
<td>8</td>
<td>4</td>
<td>10</td>
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</tbody>
</table>

Total Capacity = 24

**Global Optimum**

<table>
<thead>
<tr>
<th></th>
<th>s1</th>
<th>s2</th>
<th>s3</th>
<th>s4</th>
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<tbody>
<tr>
<td>r0</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>r1</td>
<td>10</td>
<td>7</td>
<td>6</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>r2</td>
<td>4</td>
<td>8</td>
<td>4</td>
<td>10</td>
<td>9</td>
</tr>
</tbody>
</table>

Total Capacity = 25
Key Observation

**Lemma:** If a relay node is shared by multiple source nodes, let $s_i$ be the source node with the minimum capacity. Making $s_i$ transmit to $d_i$ directly can increase the total capacity.
What the Lemma Implies

Lemma

System Model

MATCHING!
Optimal Relay Assignment

- Construct a bipartite graph
- Construct nodes corresponding to source nodes
- Construct nodes corresponding to relay nodes
- Construct nodes corresponding to destination nodes
- Set weight on the source-relay edge as the achievable capacity of CC
- Set weight on the source-dest edge as the capacity of DT
- Find the maximum weighted matching
- Assign relays according to the matching
### Source Nodes Are Selfish

#### Global Optimum

<table>
<thead>
<tr>
<th></th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
<th>$s_4$</th>
<th>$s_5$</th>
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<tbody>
<tr>
<td>$r_0$</td>
<td>4</td>
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<td>1</td>
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<td>$r_1$</td>
<td>10</td>
<td>7</td>
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<td>6</td>
<td>8</td>
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<tr>
<td>$r_2$</td>
<td>4</td>
<td>8</td>
<td>4</td>
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</table>

Total Capacity = 25

#### Nash Equilibrium

<table>
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<td>8</td>
<td>4</td>
<td>10</td>
<td>9</td>
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</tbody>
</table>

Total Capacity = 17

---

The total capacities are shown in the table above. The global optimum has a total capacity of 25, while the Nash equilibrium has a total capacity of 17.
# How Bad is Selfish Selection

<table>
<thead>
<tr>
<th></th>
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<th>$s_2$</th>
<th>$s_3$</th>
<th>$\cdots$</th>
<th>$s_{n-1}$</th>
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<tr>
<td>$r_4$</td>
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<td>1</td>
<td>5</td>
<td>$\cdots$</td>
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<td>1</td>
<td>1</td>
<td>$\cdots$</td>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

### NE

$$\frac{C(\text{NE})}{C(\text{OPT})} = \frac{5n}{10n} = \frac{1}{2}$$
Mechanism to Induce the Selfish Players

- Deviate from Optimal Strategy → Penalty
- At Optimal Strategy → No Penalty
- Payment = (capacity) + self-penalty - average-penalty of others
Payment of the Source Nodes

\[
p_i^s = \begin{cases} 
C(s_i, \gamma_i) + \left[ g(\gamma_i, \gamma_i^*) - \frac{1}{n-1} \sum_{k \neq i} g(\gamma_k, \gamma_k^*) \right], & \text{if } \gamma_i \neq r_0, \\
g(\gamma_i, \gamma_i^*) - \frac{1}{n-1} \sum_{k \neq i} g(\gamma_k, \gamma_k^*), & \text{if } \gamma_i = r_0, 
\end{cases}
\]

\[g(\gamma_i, \gamma_i^*) = l \cdot |x - y|,\]

where \( \gamma_i = r_x, \gamma_i^* = r_y, l = \max_{s_i \in S} C_{DT}(s_i) + \varepsilon, \) and \( \varepsilon > 0. \)
**THEOREM**: The optimal relay assignment **IS** the unique **Strongly Dominate Strategy Equilibrium (SDSE)**.
Incentive for Relay Nodes

Problem: Relay Nodes relay data at the cost of their own resource

A simple solution: Pay them the amount of the achieved capacity by cooperative communication

Not work! Relay nodes can cheat!
Why Cheating Matters

The actual total capacity decreases from 5 to 3

The actual total capacity decreases from 8 to 7
VCG-based payment: Each winning bidder receives the payment, which is equal to the actual achievable capacity subtracted by the opportunity cost that its presence introduces to all the other bidders.
Payment to the Relay Nodes

\[
p^r_j = \begin{cases} 
0, & \sigma_j = s_0, \\
C(\sigma_j, r_j) - \left(\Psi(S, R \setminus \{r_j\}) - \Psi(S \setminus \{\sigma_j\}, R \setminus \{r_j\})\right), & \text{otherwise}
\end{cases}
\]

where \(\sigma_j\) is the source node \(r_j\) is assigned to, and \(\Psi(S, R)\) is the system capacity of the network consisting of \(S\) and \(R\).
**HERA**

Relay nodes report their power

Source nodes select relay nodes

Administrator charges source nodes and pays relay nodes
Wrap Up of Part II

We designed HERA, an integrated optimal relay assignment scheme.

HERA induces selfish source nodes to converge to the optimal assignment.

HERA prevents relay nodes from cheating on their power.

HERA is budget-balanced.
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Motivation

Capacity demand continually grows in wireless networks

- E.g. Cellular networks

A significant amount of money has been spent on capacity enhancement

- E.g. AT&T spent approximately 19 billion dollars in 2010
Motivation
Why Auction

Relay node consumes its own resources.

CPU

Memory

Power
What is Auction

An auction is a process of buying and selling goods or services by offering them up for bid, taking bids, and then selling the item to the highest bidder. In economic theory, an auction may refer to any mechanism or set of trading rules for exchange.
Cooperative Communication Auction
## Auction Formulation

<table>
<thead>
<tr>
<th>Source Node</th>
<th>Relay Node</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bidder</strong></td>
<td><strong>Buyer</strong></td>
</tr>
<tr>
<td><strong>Private Type</strong></td>
<td><strong>Achievable Capacity</strong> ( (V_{ij}) )</td>
</tr>
<tr>
<td><strong>Utility</strong></td>
<td>( V_{ij} - P_i^b )</td>
</tr>
</tbody>
</table>
Desirable Economic Properties

- Individual Rationality
- Budget Balance
- Truthfulness
- System Efficiency

Impossible to satisfy ALL four properties.

R. Myerson  M. Satterthwaite
TASC

Truthful Auction Scheme for Cooperative Communications

- Individual rational, budget balanced and truthful
- Allow the auctioneer to choose different allocation algorithms
Challenges of Designing a Cooperative Communication Auction

Double auction
- Consider both buyers and sellers

Multiple heterogeneous items
- Each buyer has preference on different sellers

Little theoretical support
- Neither Computer Science society nor Economic society
- VCG double auction does not work
## Existing Work

<table>
<thead>
<tr>
<th>Existing Work</th>
<th>Heterogeneous Item</th>
<th>Double Auction</th>
<th>Truthful</th>
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</tbody>
</table>
TASC: Overview

Bid/Ask-independent assignment

• Achieve the truthfulness

Based on McAfee double auction

• Achieve all three economic properties while enabling multi-item auction
McAfee Double Auction

Winning buyers, each paying $4

Buyers: $8, $7, $5, $4, $3, ... ≥ ≥ ≥ ≥ <

Winning sellers, each receiving $3

Sellers: $1, $2, $3, $3, $4, ...
TASC: Design

- Construct the bipartite graph
- Apply bid/ask-independent relay assignment algorithms
TASC: Design

- Relay Assignment
- Winner Determination
- Pricing

- Apply McAfee auction

$8 \rightarrow$7 \rightarrow $5 \rightarrow $4 \rightarrow $3

$1 \rightarrow $2 \rightarrow $3 \rightarrow $3 \rightarrow $4
Charge the buyers the bid of the sacrificed buyer
Pay the sellers the ask of the sacrificed seller
## TASC: Example

### Capacity (Bid)

<table>
<thead>
<tr>
<th></th>
<th>r1</th>
<th>r2</th>
<th>r3</th>
<th>r4</th>
<th>r5</th>
<th>r6</th>
<th>r7</th>
</tr>
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<tbody>
<tr>
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### Ask

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<th>r3</th>
<th>r4</th>
<th>r5</th>
<th>r6</th>
<th>r7</th>
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<td>5</td>
<td>6</td>
<td>4</td>
<td>1</td>
<td>7</td>
</tr>
</tbody>
</table>
Winning buyer-seller pairs: (s₁, r₁) and (s₅, r₆)
Each Winning buyer pays 8 and each winning sellers receives 6
Auctioneer’s profit is 2*(8-6) = 4
Properties of TASC

- TASC is individual rational
- TASC is truthful
- TASC is budget-balanced
- Any bid-independent allocation algorithm can be applied
Conclusions

Game theory is an appropriate tool to analyze the network with independent individuals belonging to different entities.

Game theory helps with the resource allocation in cooperative networks.

Auction theory provides incentives to the individuals to participate in cooperative communication.
Results presented are joint work with my students:

- Dejun Yang
- Xi Fang
Challenges

- Utility function selection
- Existence and uniqueness of NE
- Computation of NE
- Efficiency of NE
- System efficiency in mechanism design
Q&A