



- Design & Development Forum

# Three Dimensional Localization in Wireless Sensor Networks using the Adapted Multi-Lateration Technique Considering Range Measurement Errors

G. Selda Kuruoglu\*, Melike Erol\*, **Sema Oktug\***

\*CNRL: Computer Networks Research Laboratory

Department of Computer Engineering,  
Istanbul Technical University, Turkey



**1. Motivations**

2. Localization Techniques & Problems

3. Proposed Method: AML

4. Simulation Results

5. Conclusions

# Outline

1. Motivations
2. Localization Techniques & Problems
3. Proposed Method
4. Simulation Results
5. Conclusions

# WSNs

- Medium is space, water -> wireless com.
- Several nodes (may rise up to thousands !!) :
  - Small
  - Sensing Capabilities for environment monitoring etc.
  - Low power !..
  - Low bit rate
  - Low cost (dispensable)
  - Autonomous

➤ Applications:

*Target Tracking, Military and Security Issues, Environmental Monitoring, Health, Home, Space Exploration, Chemical Processing, Disaster Relief, etc...*

➔ 3D Localization is essential in most of them!..

## Localization Problem:

- ❖ Nodes are usually spread out in environment  
(*deployment phase*)

## Localization Problem:

- ❖ Nodes are usually spread out in environment (*deployment phase*)
- Localization: estimating the location of a node
  - **Global:** latitude, longitude, altitude
  - **Local:** position information relative to other nodes

## Localization Problem:

- ❖ Nodes are usually spread out in environment (*deployment phase*)
- Localization: estimating the location of a node
  - **Global**: latitude, longitude, altitude
  - **Local**: position information relative to other nodes
- Why location information is needed ??

## Localization Problem:

- ❖ Nodes are usually spread out in environment (*deployment phase*)
- Localization: estimating the location of a node
  - **Global**: latitude, longitude, altitude
  - **Local**: position information relative to other nodes
- Why location information is needed ?? *data tagging*
- Location info. can also be used in *position-aware routing algorithms*

# Some 'Localization' Terms:

- **Physical Position:** numeric coordinate system
- **Symbolic Location:** Relative or application related info
- **Anchor**  $\equiv$  **beacon**  $\equiv$  **landmark:** know their own position
- **Localized vs Centralized Computation**
  - Computations are done by? the nodes or a common center
- **Accuracy, Precision**
- **Range-free :** proximity based
- **Range-based: precise distant measurements**
- **Single/Multi Hop**
- **Iterative**

## Range- Based Localization Schemes:

- Uses distance measurements retrieved by:

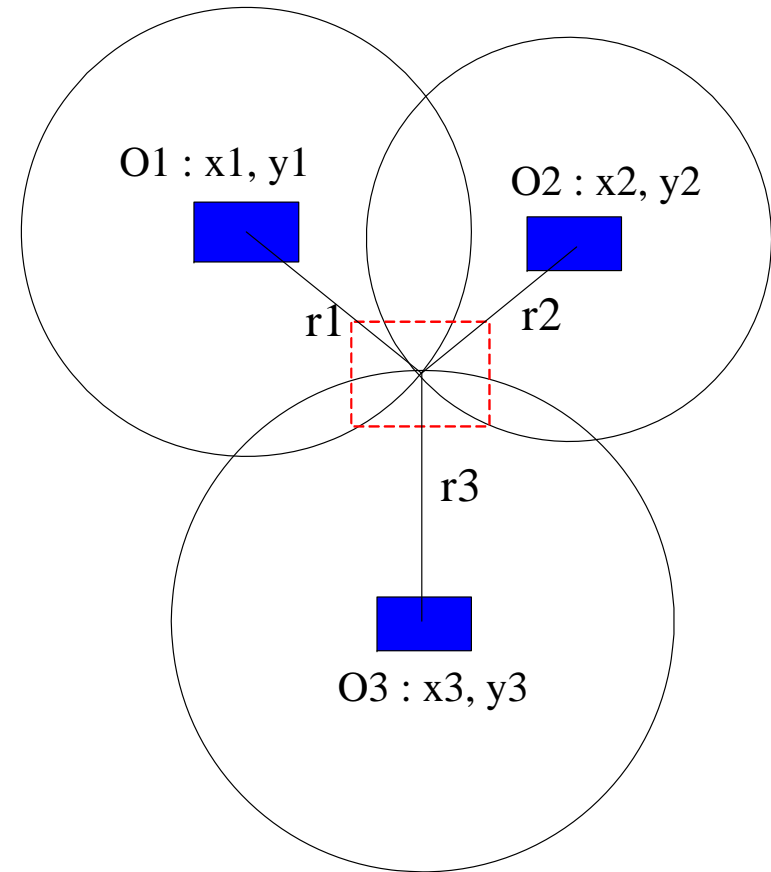
❖ RSSI	: Received Signal Strength Indicator
❖ ToA	: Time of Arrival
❖ TDoA	: Time Difference of Arrival
❖ AoA	: Angle of Arrival
❖ Dv Hop	: Approximation...

*What about  
measurement  
errors ??*

- Multi-Lateration\*
- Bounding-Box
- APIT (Approximate point in triangle)
- Arithmetic (Blusu et al, GPS free localization)

## How to Calculate Location?? (Range-Based)

- **Lateration, Multi – Lateration (2D demonstration)**
  - Measure distances from 3 anchors
  - Possible location points form a circle
  - the intersection of those 3 circles: real loc.
  - Used for performance comparisons of AML

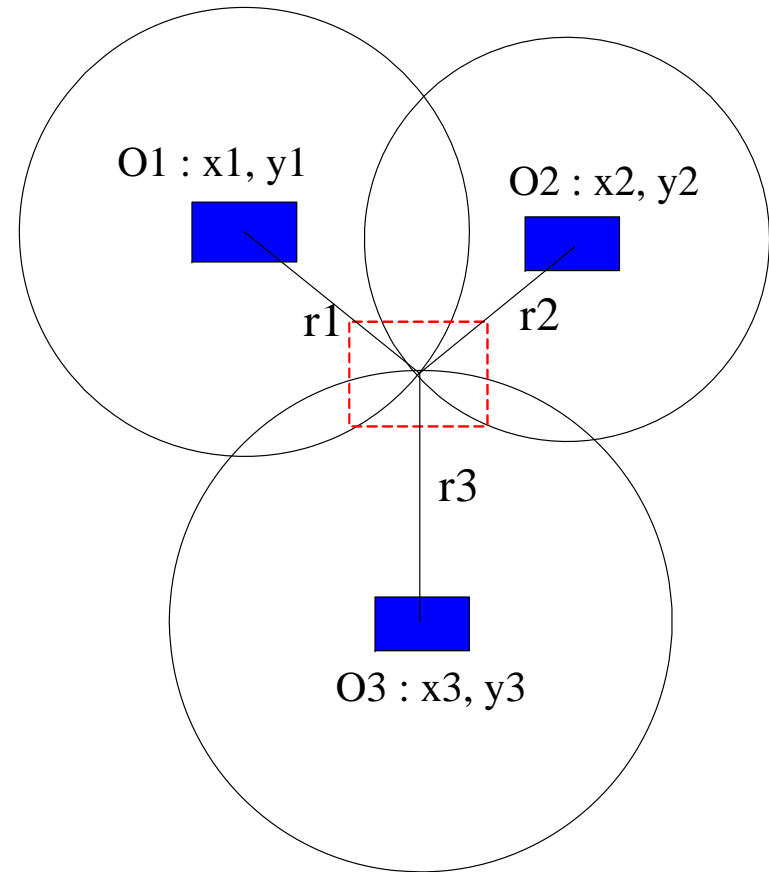


## How to Calculate Location?? (Range-Based)

- **Lateration, Multi – Lateration (2D demonstration)**
  - Measure distances from 3 anchors
  - Possible location points form a circle
  - the intersection of those 3 circles: real loc.
  - Used for performance comparisons of AML

### Problem :

which one if the the intersection will not result in a single point → **multilateration** ( more distance measurements )



# Multi-Lateration with Distance Measurement Errors:

Sphere Equations:

$$\begin{aligned}
 (x_1 - x_U)^2 + (y_1 - y_U)^2 + (z_1 - z_U)^2 &= r_1^2 & \text{I} \\
 (x_2 - x_U)^2 + (y_2 - y_U)^2 + (z_2 - z_U)^2 &= r_2^2 & \text{II} \\
 (x_3 - x_U)^2 + (y_3 - y_U)^2 + (z_3 - z_U)^2 &= r_3^2 & \text{III} \\
 (x_4 - x_U)^2 + (y_4 - y_U)^2 + (z_4 - z_U)^2 &= r_4^2 & \text{IV}
 \end{aligned}$$

I-IV:  $(x_1 - x_U)^2 - (x_4 - x_U)^2 + (y_1 - y_U)^2 - (y_4 - y_U)^2 + (z_1 - z_U)^2 - (z_4 - z_U)^2 = r_1^2 - r_4^2$

...

$$2(x_4 - x_1)x_U + 2(y_4 - y_1)y_U + 2(z_4 - z_1)z_U = (r_1^2 - r_4^2) - (x_1^2 - x_4^2) - (y_1^2 - y_4^2) - (z_1^2 - z_4^2)$$

II-IV:  $(x_2 - x_U)^2 - (x_4 - x_U)^2 + (y_2 - y_U)^2 - (y_4 - y_U)^2 + (z_2 - z_U)^2 - (z_4 - z_U)^2 = r_2^2 - r_4^2$

...


$$2(x_4 - x_2)x_U + 2(y_4 - y_2)y_U + 2(z_4 - z_2)z_U = (r_2^2 - r_4^2) - (x_2^2 - x_4^2) - (y_2^2 - y_4^2) - (z_2^2 - z_4^2)$$

III-IV:

$$(x_3 - x_U)^2 - (x_4 - x_U)^2 + (y_3 - y_U)^2 - (y_4 - y_U)^2 + (z_3 - z_U)^2 - (z_4 - z_U)^2 = r_3^2 - r_4^2$$

...

$$2(x_4 - x_3)x_U + 2(y_4 - y_3)y_U + 2(z_4 - z_3)z_U = (r_3^2 - r_4^2) - (x_3^2 - x_4^2) - (y_3^2 - y_4^2) - (z_3^2 - z_4^2)$$

Generalize for more anchors,  
Rewrite as a linear matrix equation: 

# Generalized Form:

$$2 \begin{pmatrix} x_n - x_1 & y_n - y_1 & z_n - z_1 \\ \dots & \dots & \dots \\ x_{n-1} - x_1 & y_{n-1} - y_1 & z_{n-1} - z_1 \end{pmatrix} \begin{pmatrix} x_U \\ y_U \\ z_U \end{pmatrix} - \begin{pmatrix} (r_1^2 - r_n^2) - (x_1^2 - x_n^2) - (y_1^2 - y_n^2) - (z_1^2 - z_n^2) \\ \dots \\ (r_{n-1}^2 - r_n^2) - (x_{n-1}^2 - x_n^2) - (y_{n-1}^2 - y_n^2) - (z_{n-1}^2 - z_n^2) \end{pmatrix}$$

$\underbrace{\hspace{10em}}_A \quad \underbrace{\hspace{1em}}_x \quad \underbrace{\hspace{10em}}_b$

$$= \begin{pmatrix} 0 \\ \dots \\ 0 \end{pmatrix}$$

error vector

$$\begin{aligned} (\|Ax-b\|_2)^2 &= (Ax-b)^T (Ax-b) \\ &= x^T A^T Ax - 2x^T A^T b + b^T b \end{aligned}$$

$$2A^T Ax - 2A^T b = 0 \Leftrightarrow \boxed{A^T Ax = A^T b}$$

Minimize the total error -> minimize norm 2 of error\_vec

known as LSE problem.

## 3D-AML: Three Dimensional Adapted Multilateration

- Aim: reduce the computational cost, increase accuracy
- uses beacon spheres as well (range-based)

### Method Overview:

1. First step: Arbitrarily pick three intersecting spheres  
Gives two/one intersection points

## 3D-AML: Three Dimensional Adapted Multilateration

- Aim: reduce the computational cost, increase accuracy
- uses beacon spheres as well (range-based)

### Method Overview:

1. First step: Arbitrarily pick three intersecting spheres  
Gives two/one intersection points
2. Choose one of the intersection points

## 3D-AML: Three Dimensional Adapted Multilateration

- Aim: reduce the computational cost, increase accuracy
- uses beacon spheres as well (range-based)

### Method Overview:

1. First step: Arbitrarily pick three intersecting spheres  
Gives two/one intersection points
2. Choose one of the intersection points
3. Move towards next (forth) sphere

## 3D-AML: Three Dimensional Adapted Multilateration

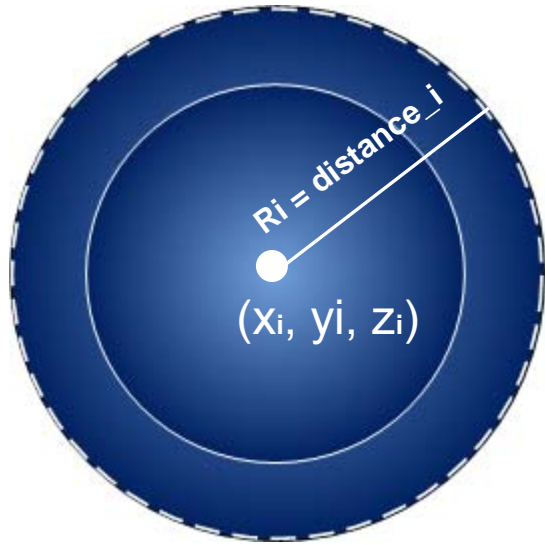
- Aim: reduce the computational cost, increase accuracy
- uses beacon spheres as well (range-based)

### Method Overview:

1. First step: Arbitrarily pick three intersecting spheres  
Gives two/one intersection points
2. Choose one of the intersection points
3. Move towards next (forth) sphere
4. Try to refine position estimation with leaving beacons

AML is implemented in three steps: *intersection and elimination*, *first estimation* and *refinement*.

# "Intersection & Elimination" Phase



- Start with intersecting 3 spheres.
- Why not two? This gives circle equation, latter gives two intersection points

Anchor  $i$

(at  $(x_i, y_i, z_i)$ ):

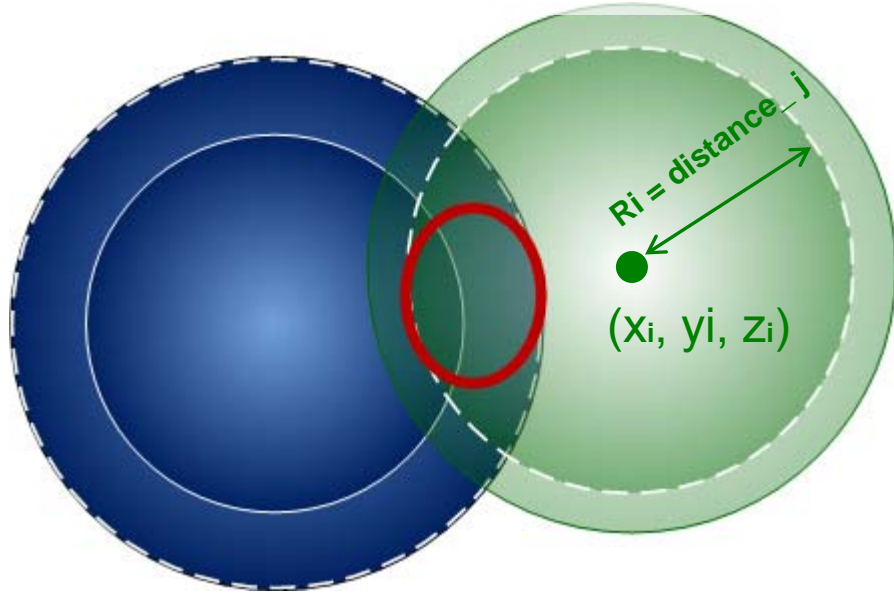


REAL Dist.s



MEASURED Dist.s

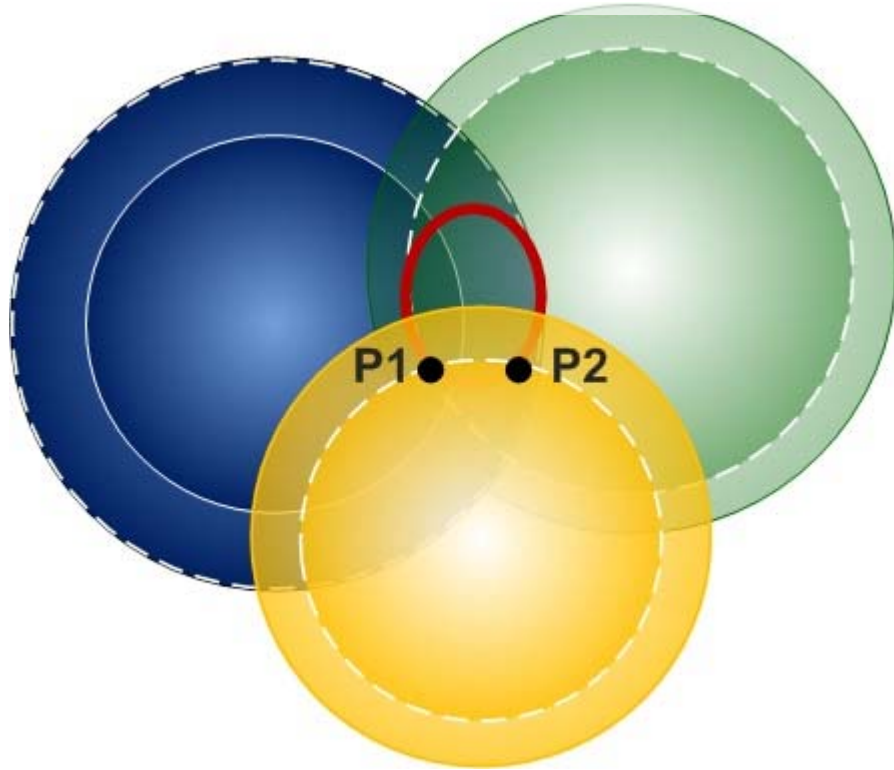
# “Intersection & Elimination” Phase



Anchor  $j$  (at  $(x_j, y_j, z_j)$ )

Intersection of two sphere surfaces randomly chosen from the anchor set : circle in red

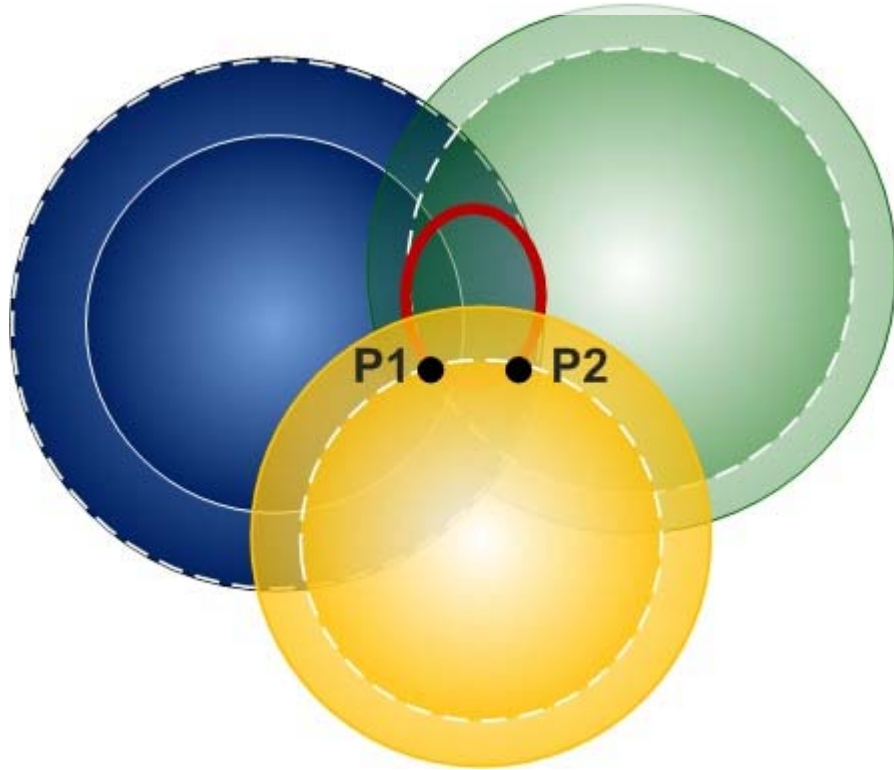
# “Intersection & Elimination” Phase



Intersection of three sphere surfaces : P1 & P2  
( Intersection phase is over... )

Anchor k (at  
( $x_k, y_k, z_k$ ))

# “Intersection & Elimination” Phase



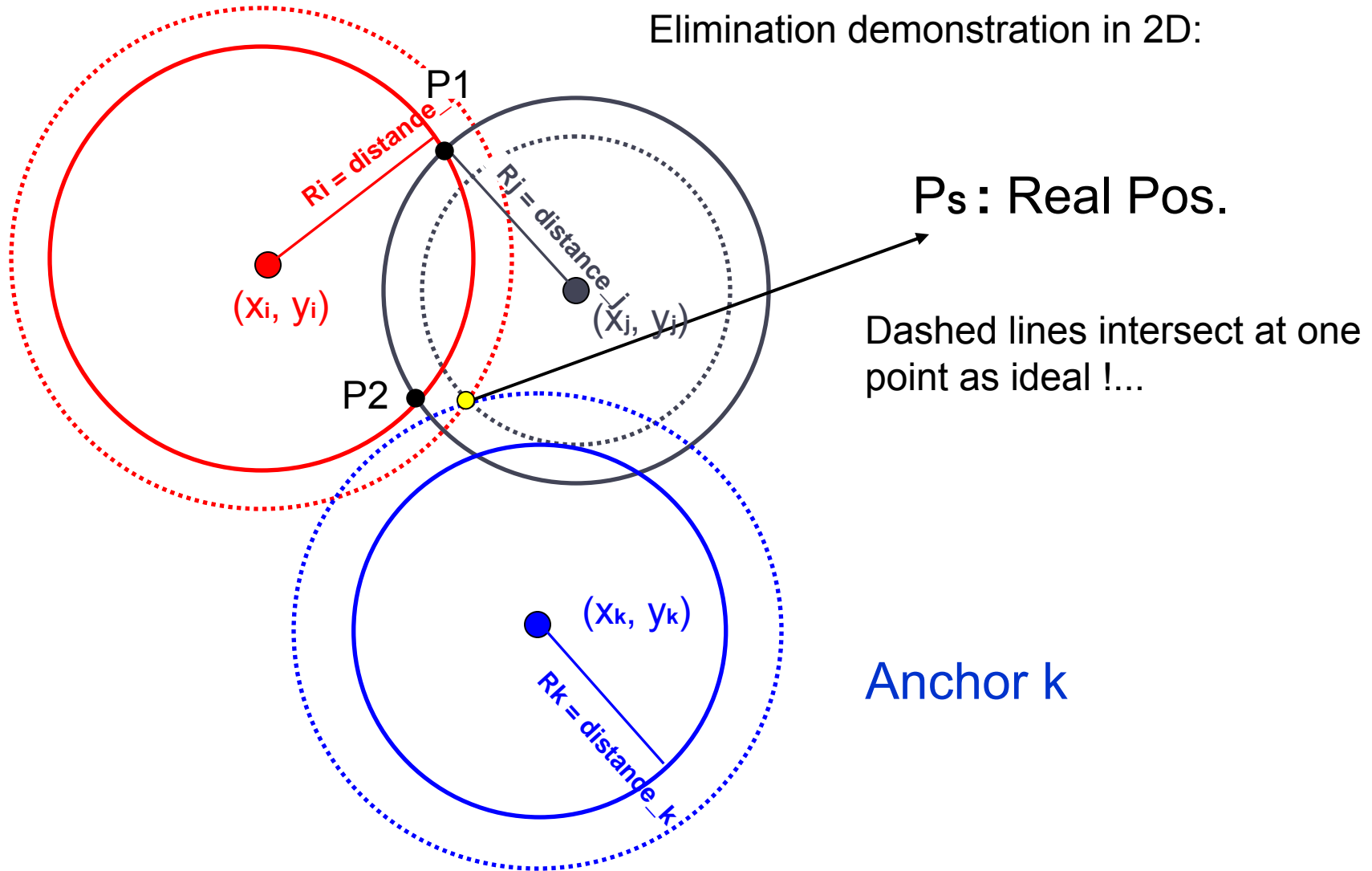
at elimination phase we have to choose between

P1 & P2:

*Principle:* Choose nearest one to the next anchor

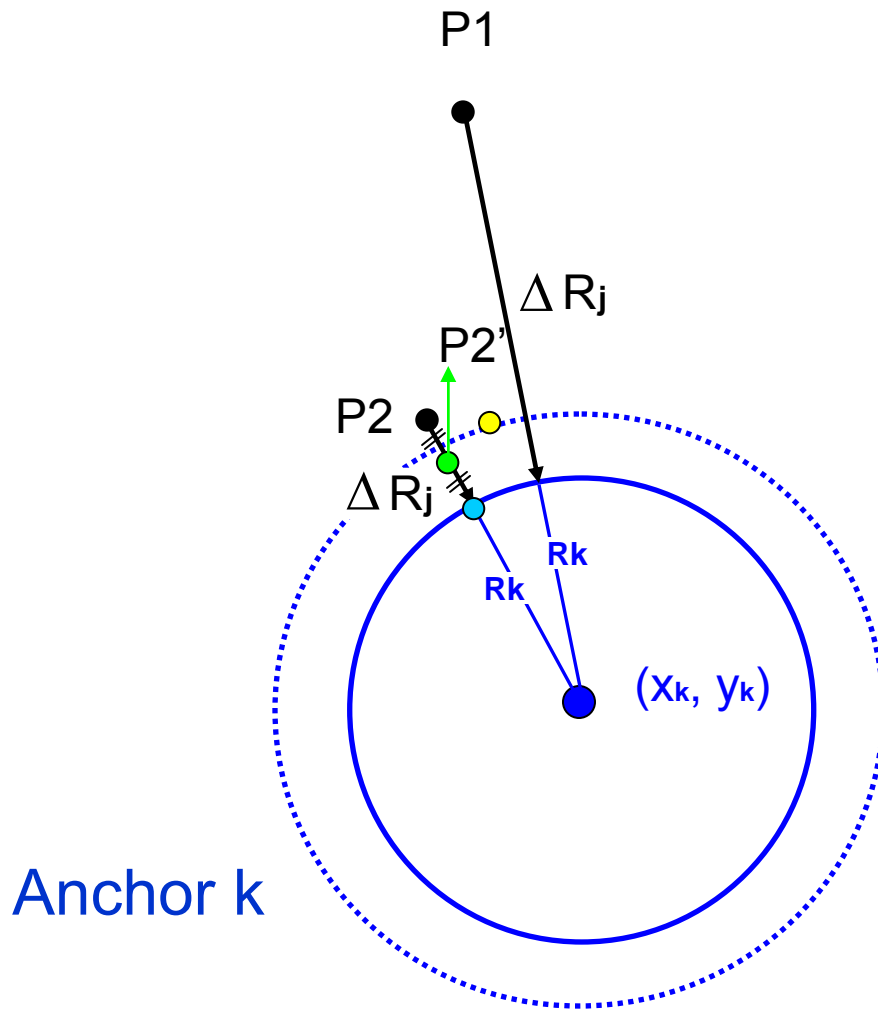
# "Intersection & Elimination" Phase

Elimination demonstration in 2D:

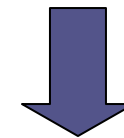




# “First Estimation” Phase

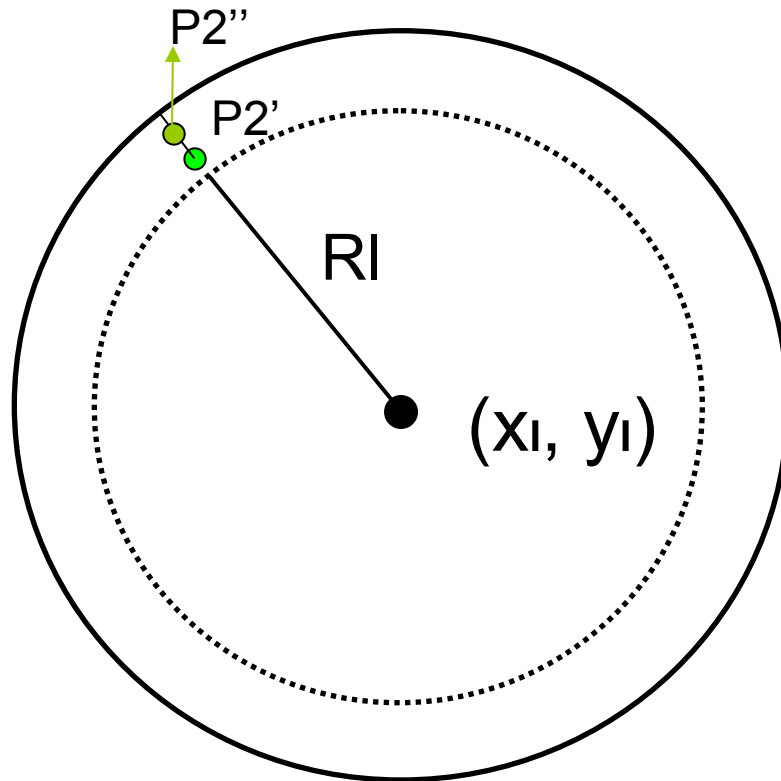


- P2 is selected.
- “ Move p. to the middle of the line connecting p. and the nearest point to p. (light blue p.) on the third circle ”  
*(sphere surface in 3D)*



MOVE P2 to P2'

# “Refinement” Phase/s



Anchor I

- Use leaving beacons.
- Same principle
- I.e. Use 4th beacon:  
Move  $P2'$  to  $P2''$

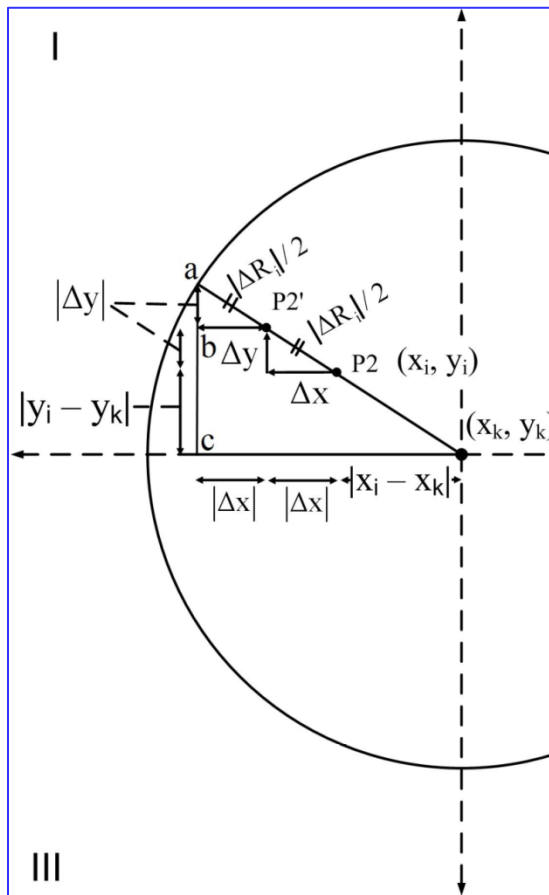
Notes:

- $P2'$  is inside beacon circle
- $\Delta RI$  is negative

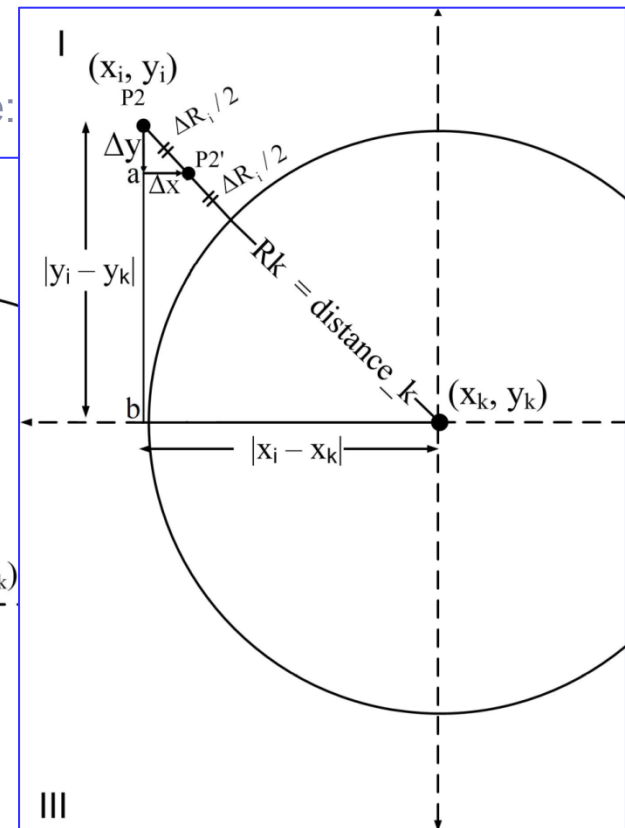
# Movement Formulations:

- Movement formulations differ when point is inside/outside beacon circle/sphere
- Besides, sign of the displacement differs in 4 quarters in 2D, 8 quarters in 3D defined from the center of the considered circle/sphere

P2, so P2' are inside k<sup>th</sup> circle:



P2, so P2' are outside k<sup>th</sup> circle:



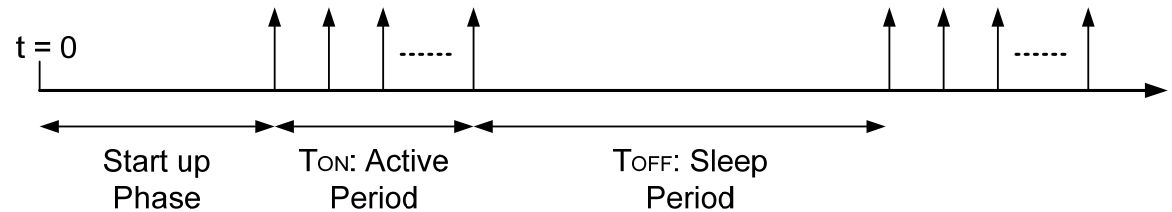
# Movement Formulations:

Point is inside the sphere:	Point is outside the sphere:
$RR = \frac{ \Delta R_i }{\Delta R_i + R_k}$ $\frac{ \Delta_x }{ x_k - x_i } = RR \Rightarrow  \Delta_x  = RR *  x_k - x_i $ $\frac{ \Delta_y }{ y_k - y_i } = RR \Rightarrow  \Delta_y  = RR *  y_k - y_i $ $\frac{ \Delta_z }{ z_k - z_i } = RR \Rightarrow  \Delta_z  = RR *  z_k - z_i $	$RR = \frac{ \Delta R_i }{R_k}$ $\frac{ \Delta_x }{2 *  \Delta_x  +  x_k - x_i } = RR \Rightarrow  \Delta_x  = \frac{RR *  x_k - x_i }{1 - (2 * RR)}$ $\frac{ \Delta_y }{2 *  \Delta_y  +  y_k - y_i } = RR \Rightarrow  \Delta_y  = \frac{RR *  y_k - y_i }{1 - (2 * RR)}$ $\frac{ \Delta_z }{2 *  \Delta_z  +  z_k - z_i } = RR \Rightarrow  \Delta_z  = \frac{RR *  z_k - z_i }{1 - (2 * RR)}$
<p><math>\Delta_x, \Delta_y, \Delta_z</math> 's sign: identical to the sign of  <math>!(x / y / z_k - x / y / z_i)</math></p>	<p><math>\Delta_x, \Delta_y, \Delta_z</math> 's sign: identical to the sign of  <math>(x / y / z_k - x / y / z_i)</math></p>

# Simulator Design

## ➤ C++, Discrete Event Simulator

3 Event Types: *sleep, wake up, try localization*



## ➤ Iterative Loc.

## ➤ Beacon usage limitation to 5 in AML-5

## ➤ Error\_percentage in error model: 20%,30%,40%

(Gaussian error modelling with zero mean and variance= $\text{realDistance} * \text{errorMaxPercentage}$ )

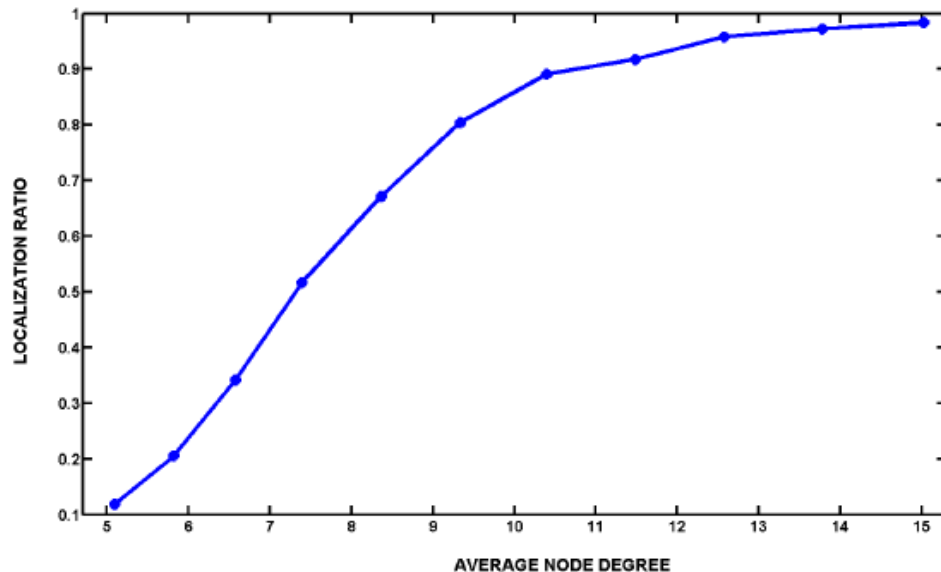
## ➤ Each point represents 10 sims at same environmental cond. with different rng seeds

## Environment:

- 200 sensor nodes in a terrain of  $100m * 100m * 100m$  (uniform randomly deployed)
- Initial beacon percentage is %20
- RR varied from 20m to 30m with 1m (affects node degree!)
- 2 schemes in 3 environments were compared:
  - Schemes:
    - Multilateration (all and 5 beacons used)
    - AML (all and 5 beacons used)
  - Environments:
    - error percentages: 20, 30 and 40%

## *Localization ratio:*

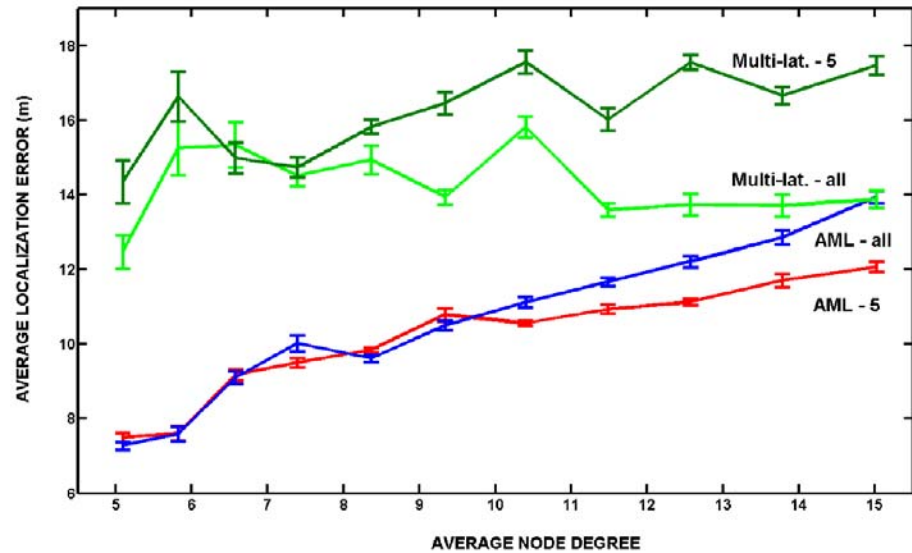
ratio of localized nodes over initial non-beacon nodes



*As node degree increases, prob. of anchor neighbor rises*

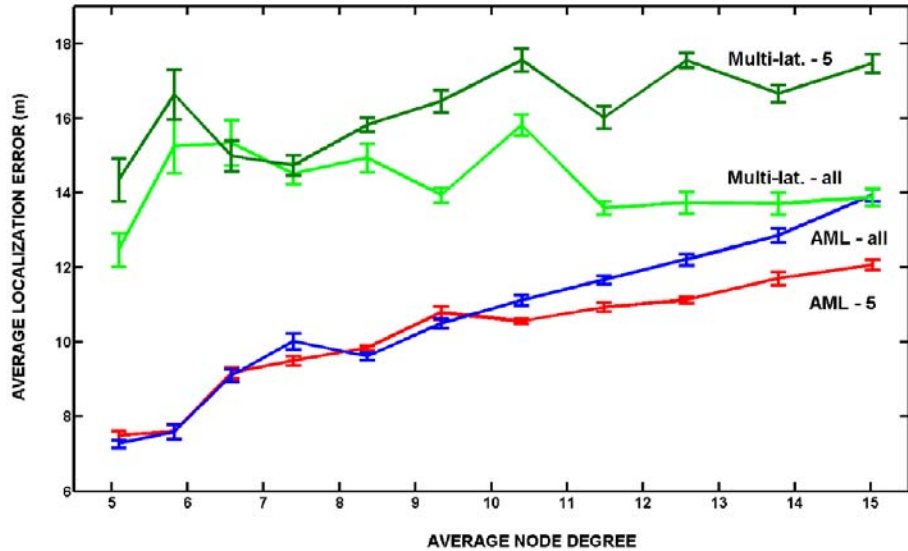
# Localization error: Mean of location estimation errors

errorPercentage: 20%

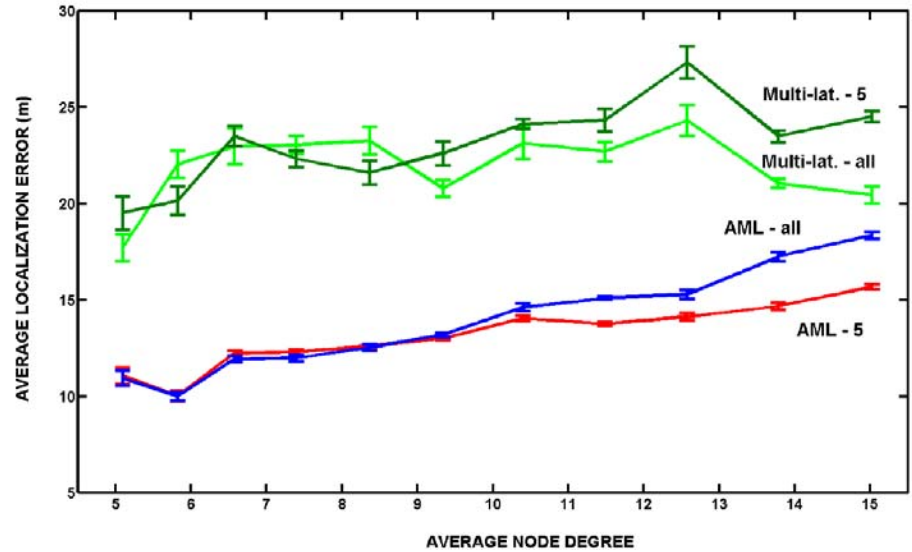


# Localization error: Mean of location estimation errors

errorPercentage: 20%

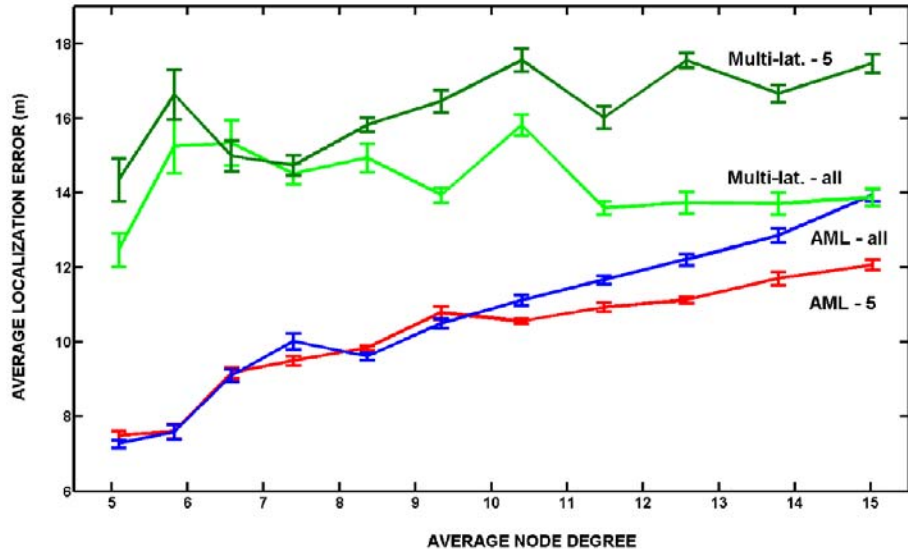


errorPercentage: 30%

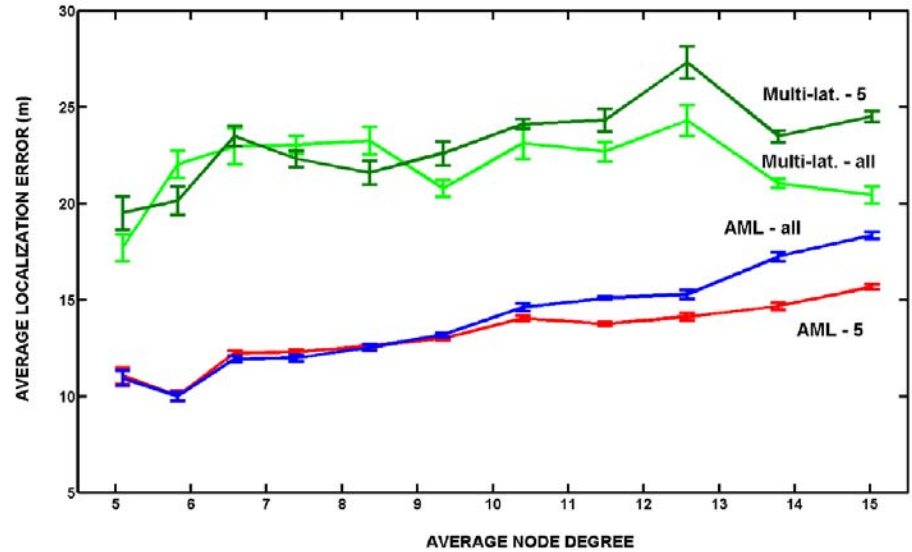


# Localization error: Mean of location estimation errors

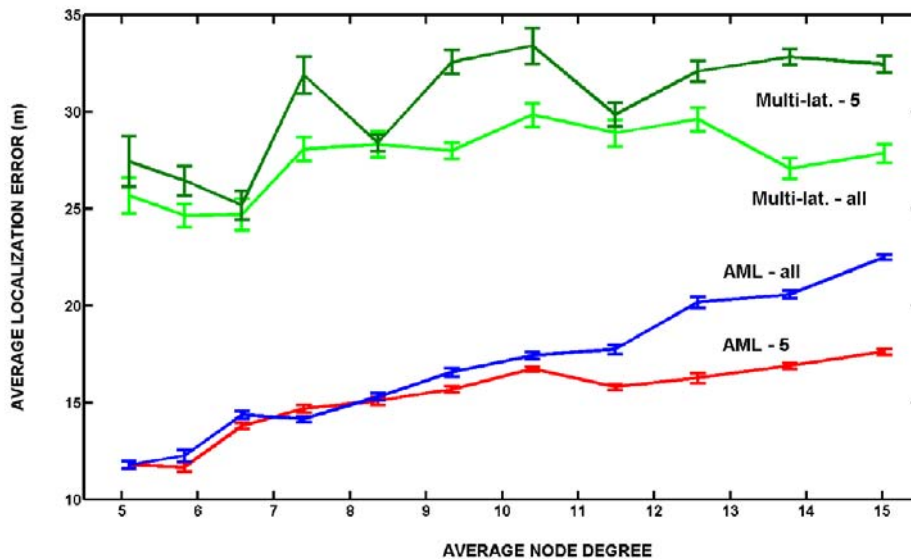
errorPercentage: 20%



errorPercentage: 30%

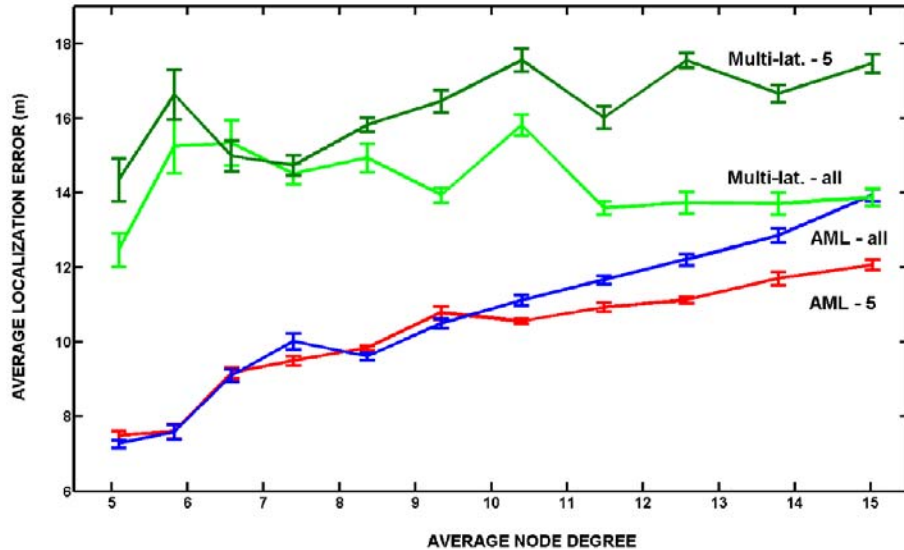


errorPercentage: 40%

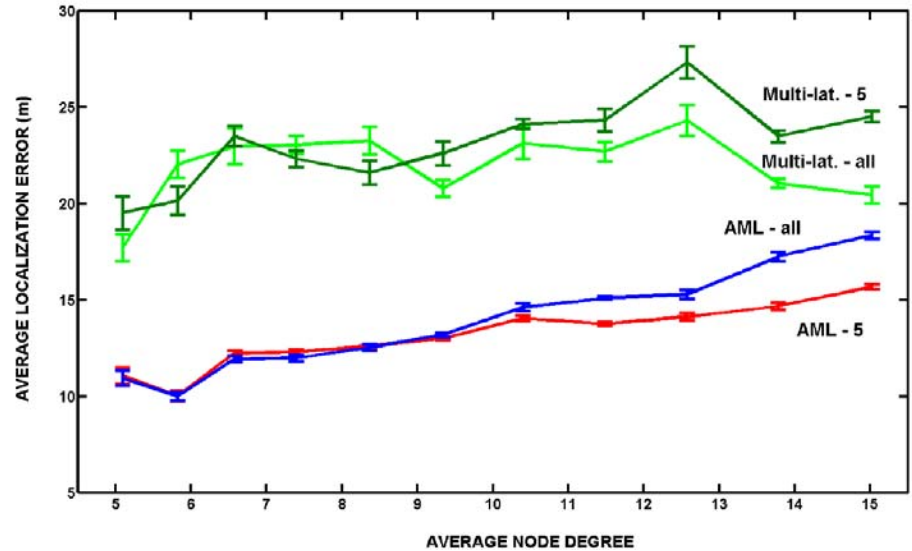


# Localization error: Mean of location estimation errors

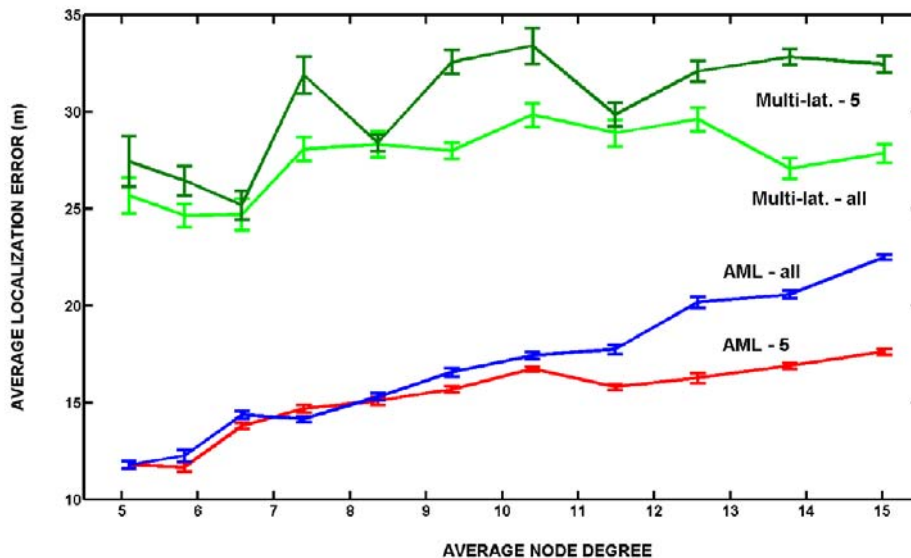
*errorPercentage: 20%*



*errorPercentage: 30%*



*errorPercentage: 40%*



- Anchor usage: favor for multilateration but not for AML at higher node degrees
- *errorPercentage* increases loc errors in all schemes..

- sensor technology ↑ => more capable nodes => Wide Usage Area
- “Localization” : key concept for interpreting the gathered data
- 3D version of a new approach named AML is developed and tested for loc. problem

## AML:

- has lower computational cost
- More robust (avg loc error)
- Disadvantage comes from heuristics (1st step)
- Can be used when GPS is not available but have some distance measurements

# Any Questions ?

## THANKS ...

Contact info for the authors:

G. Selda KURUOĞLU, [kuruoglug@itu.edu.tr](mailto:kuruoglug@itu.edu.tr)

Melike EROL, [melike.erol@itu.edu.tr](mailto:melike.erol@itu.edu.tr)

Sema Oktug, [oktug@itu.edu.tr](mailto:oktug@itu.edu.tr)